



PREDICTION OF HYDRODYNAMIC BEARING PERFORMANCE BASED ON EFFECTIVE PARAMETERS BY NEURAL NETWORK

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ABSTRACT

Condition monitoring of rotary device is one of the major concerns which all industries have followed the new method for preventing unpredicted failure. In this way predictive maintenance (PM) is the most significant part of our industries. In this work, the main objective is to analyze the variation of three parameters including, Shear stress, Power losses and RMS (as an acoustic emission feature) on hydrodynamic journal bearing under different lubricants for various loading conditions and various rotational speeds. The results obtained experimentally from variation of parameters at different levels lead to find the relationship between output parameters and input factors. Artificial neural network (ANN) by using multilayer perceptron algorithm is applied to process the set of large number data from the test, with 80% used for training and 20% used for testing the predicted model. In addition to this, 20% of real data have been applied for test of the mentioned network. The accuracy of predicted model is about 0.001. The results show that the presented model from neural networks, constituting methodical basis for the control and diagnostics the bearing without prior knowledge of the relative rotational speeds or load conditions can be predicted with reasonable accuracy which hitherto has not been explored. Also, this method can utilize for the Interpolation of parameters which cannot be tested in real condition for the assessment of behavior of output parameters.

KEYWORDS: Hydrodynamic Bearing, Acoustic Emission, Multi-layer perceptron (MLP)

INTRODUCTION

To reduce unplanned maintenance of rotary machine, some of the highlighted techniques of condition monitoring have been considered in various industries. The objective of these techniques is to maximize equipment performance as well minimizing condition monitoring costs. They are lots of advanced technologies in order to determine equipment condition, and potentially predict failure. It includes, but is not limited to, technologies such as: vibration measurement and analysis [1], infrared thermography [2], oil analysis and tribology [3], ultrasonic and acoustic emission [5,6].

Vibration analysis tools, such as accelerometers and eddy current sensors, represent the most common methods to diagnose bearings [5]. Infrared thermography [2] has been also investigated in recent years, but the sensors are hard to install and the tests are hard to carry out. The above mentioned studies are representative techniques to show the rising interest in using novel methods and algorithms to perform better diagnosis of bearing faults in order to develop effective prognostic tools. In this way acoustic emission can play one of the key roles for this target. Due to the fact that it can utilize for detecting defect in rotary machine for preventing sudden

failure. Acoustic Emission technique, famous for its sensitivity in high frequency domain of micro-damage evolution, has been found in monitoring of rotary machine such as: Hydro dynamic bearing. The traditional definition of Acoustic Emission (AE) is an elastic wave produced as a result of swift discharge of energy from a source within a material that is compelled by an externally applied stimulus [7, 8].

The first investigation of phenomenon of acoustic emission was accomplished by Kaiser [9]. Kaiser was the first to digitally acquire AE signals produced in the crystal structure of materials during stress tests. Advantages of acoustic emission in comparison to other diagnostic methods such as vibration analysis is to offer early defect detection. However it is difficult in interpreting and classifying the acquired information [9,10].

From the studies [11,17] it can be found that the hydrodynamic bearing is used in high speed rotary device. In point of cost issue, this type of bearing in comparison to other part of rotary device is inexpensive. In the other hand, because of its material properties, the prior defect will be existed on the bearing before it develops and affects other parts of rotary

device which are more expensive and sensitive. The traditional methods for fault diagnosis are categorized as pattern classification, knowledge-based inference and numerical modeling. Pattern classification and knowledge-based inference techniques are used in the industry. For both of the mentioned methods, a human expert looks for particular patterns in the vibration signature that might indicate the presence of a fault in the bearing. Alternatively, statistical analysis and Artificial Neural Networks (ANNs) are utilized for the automated fault detection systems. ANNs are capable of learning the behavior of nonlinear systems one of the first applications of ANNs for bearing fault diagnosis has been proposed by Baillie and Mathew [11]. Recently, Mirhadizadeh and Mba [12,13] investigated the wearing defect in hydrodynamic bearings at different condition by AE. The ability of AE method has been concluded in this research perfectly. In addition to this, some of the mechanical parameters can be applied as parallel techniques to expedite our analysis for detaching defect in rotary machine. In this work, some of the highlighted parameters such as shear stress and power loss have been deliberated as mechanical parameters for condition monitoring in hydrodynamic bearing. The set-up was made to understand the factors such as AE r.m.s, shear stress and power loss as condition monitoring of hydrodynamic bearing. The relationship between AE r.m.s and shear stress and also AE r.m.s and Power loss have been considered by Mirhadzideh and M.ba [11,12]. In this work results have been

confirmed by the practical investigation. The aim of this investigation was to predict and interpolate a test range that was not practically experienced. In order to this, the desired purpose a method which is named Multi-layer perceptron (MLP) has been utilized for prognosticating of shear stress and power loss and A.E (R.M.S) as effective parameters for condition monitoring of above mentioned parameters.

NEURAL NETWORK

Neural Networks is a field of Artificial Intelligence (AI) where we, by inspiration from the human brain, find data structures and algorithms for learning and classification of data. Many tasks that humans perform naturally fast, such as the recognition of a familiar face, proves to be a very complicated task for a computer when conventional programming methods are used. By applying Neural Network techniques a program can learn by examples, and create an internal structure of rules to classify different inputs, such as recognizing images. Learning with MLP neural networks Building on the algorithm of the simple Perceptron, the MLP model not only gives a perceptron structure for representing more than two classes, it also defines a learning rule for this kind of network.

The MLP is divided into three layers: the input layer (load, Speed, Oil type), the hidden layer and the output layer (shear stress, Power loss, AE r.m.s), where each layer in this order gives the input to the next. The extra layers give the structure needed to recognize non-linearly separable classes.

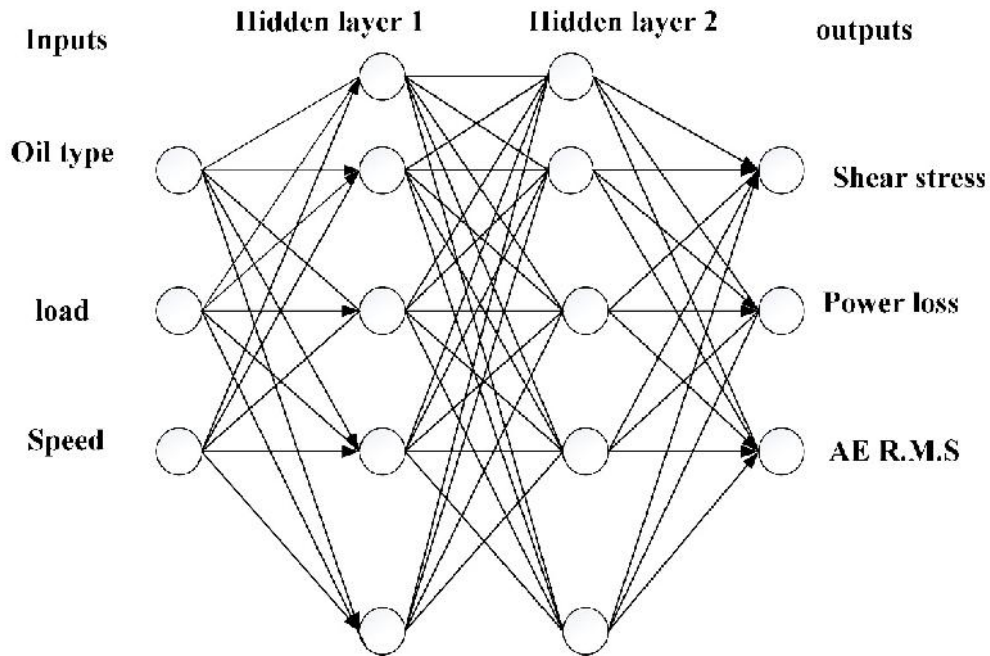


Fig.1.the structure of applied neural network

The threshold function of the units is modified to be a function that is continuous derivative, the Sigmoid function (Formula

1 The Sigmoid Function). The use of the sigmoid function gives the extra information necessary for the network to

implement the back-propagation training algorithm. Back-propagation works by finding the squared error (the Error function) of the entire network, and then calculating the error term for each of the output and hidden units by using the output from the previous neuron layer. The weights of the entire network are then adjusted with dependence on the error term and the given learning rate. Training continues on the training set until the error function reaches a certain minimum. If the minimum is set too high, the network might not be able to correctly classify a pattern. But if the minimum is set too low, the network will have difficulties in classifying noisy patterns

Eq.1

$$y_k^1 = \frac{1}{1 + e^{-w^{1kT} x - a_k^1}}, k = 1, \dots, M_1$$

$$y^1 = (y_1^1, \dots, y_{M_1}^1)^T$$

$$y_k^2 = \frac{1}{1 + e^{-w^{2kT} y^1 - a_k^2}}, k = 1, \dots, M_2$$

$$y^2 = (y_1^2, \dots, y_{M_2}^2)^T$$

...

$$y_{out} = F(x; W) = w^{pT} y^{p-1}$$

Eq.2

$$(x^1, y_1), (x^2, y_2), \dots, (x^N, y_N)$$

Eq.3

$$E(t) = (y(t)_{out} - y_t)^2 = (F(x^t; W) - y_t)^2$$

EXPERIMENTAL PROGRAM

HYDRODYNAMIC BEARING

The hydrodynamic bearing test rig employed for this study has an operational speed range between 30rpm to 5000rpm with a maximum load capability of 20kN. The hydrodynamic bearing material was Bronze. The test bearing had a radius of 17.5mm, length of 63mm, a surface roughness of approximately 3 μ m and a measured radial clearance of 0.075mm.

Oil

To understand the influence of viscosity on generating AE signals, three different oil types were employed. The property of oil types is given in table 1.

μ_d kinematic viscosity (Cst)			
Temp & Oil	SAE40	SAE68	20W50
40	186	110	160
100	14.5	11	17

Table1. The property of oil types and Viscosity

AE Sensor

Acoustic emission software AE Win and a data acquisition system (PAC) PCI-2 with a maximum sampling rate of 40 MHz were used to record AE events. A broadband, resonant type, single-crystal piezoelectric transducer from physical Acoustic Corporation (PAC), called PICO, was used as the AE sensor. The sensor had a resonance frequency of 513.28 kHz and an optimum operating range of 100-750 kHz. Both sensors were placed directly onto the test bearing housing. The surface of the sensor was covered with grease to provide good acoustic coupling between the bearing housing and the sensor. The signal was detected by the sensor and enhanced by a 2/4/6-AST pre-amplifier. The gain selector of the preamplifier was set to 40 dB. The test sampling rate was 2 MHz with 16 bits of resolution between 10 and 100 dB. AE signals were captured during the tests. Signal descriptors, such as amplitude, duration, rise time, counts, and energy, were calculated by the AE software (AE Win).

THERMOCOUPLES

One LM35-type thermocouple was fixed on the bearing housing. The thermocouple had an operating range of -55 to +150°C with an accuracy of $\pm 1.0^\circ\text{C}$. Measurements of temperature were taken throughout all test conditions at a sampling rate of 50 Hz.

EXPERIMENTAL SET-UP

This investigation involved varying the bearing rotational speed for fixed loads and also investigating the influence of different viscosities whilst recording the AE RMS levels as well as analyzing of power loss and shear stress. Prior to performing the tests, the temperature of the test bearing was raised to approximately 60°C. As soon as the desired temperature was reached, the test sequence began. In developing and understanding the relationship between Acoustic Emission and the operational variables, design of experiments was developed which involved testing at a speed range of 750, 1500, 2500, 3500, and 4500rpm and a load range of 2,6,10N carried out by pneumatic cylinder. Also the effect of three oil types (SAE40, SAE68, 20W50) in this work was investigated. In addition, the drive motor (1.5 Hp) was connected to the drive shaft via a belt drive thus eliminating

AE noise generated from the electric motor. Silicon plastics were used to eliminated noise which have been occurred between shaft and bearing (P-202). Acquisition of AE and

temperatures values were acquired continuously over duration of 5 sec. for every speed, viscosity and load condition tested.

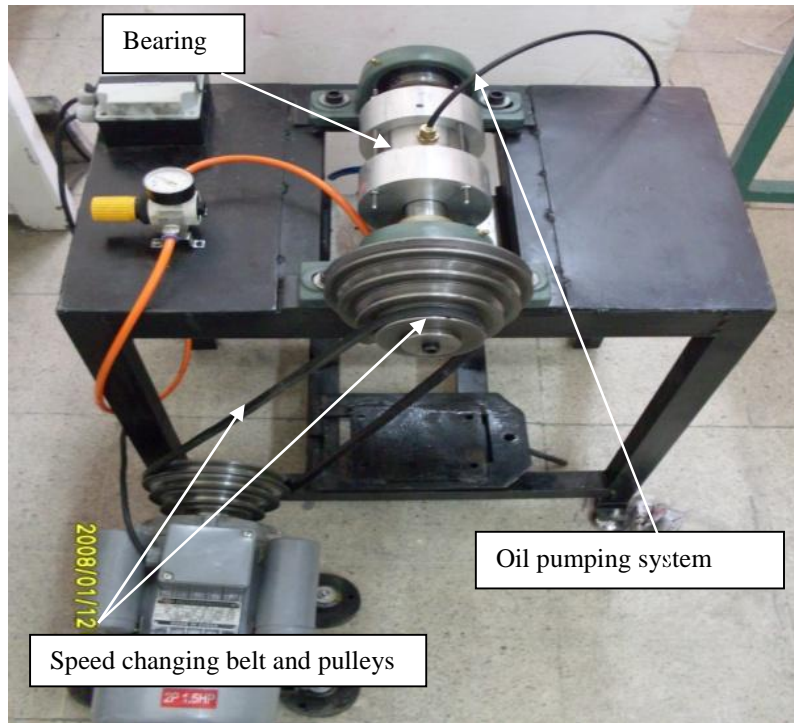


Fig.2. Test set-up

RESULT AND DISCUSSION

The condition monitoring of a bearing is judged by symptoms and signs, which are generally related to the operation parameters. The variation in time of these parameters is an indicator of the fault progression and can be used to forecast the future trend of its development, as well as serving as the basis for generating alarm signals. Neural networks can "learn" by adjusting the parameters such that certain input signals correspond to a desired response. Such a "training" process is a continuous process until no significant adjustment is required.

Experiments were designed for three oil at different viscosity, three transverse load and five rotating speeds on a hydro dynamic bearing. In this way, the variation of three output parameters i.e. Shear stress, Power losses and RMS (as an acoustic emission feature) on hydrodynamic journal bearing is measured.

The experimental results provided input to the design and configuration of a suitable neural network, which was then used to provide an accurate description of the bearing condition in an on-line fashion. To evaluate the performance of a neural network for fault diagnosis, a combination of various parameters needs to be considered. These include the activation functions and a suitable learning rate. A neural network architecture for estimation of bearings condition has been illustrated in figure 1, using a MLP. Figure 1 shows that

each input from the input layer is fed up to each node in the hidden layer, and from there to each node on the output layer. We should note that there can be any number of nodes per layer and there are usually multiple hidden layers to pass through before ultimately reaching the output layer. Choosing the right number of nodes and layers is important later on when optimizing the neural network to work well a given problem. In this study, networks of different configurations (different input and hidden layer sizes) were trained with the data set produced by the experiments. The objective was to arrive at a combination of input parameters for a neural network that can interpolate the results without prior knowledge with the least error.

Complexity can be built into a neural network by adding hidden layers between the input and output layers. Such hidden layers help in modeling the non-linear behavior of the system. However, adding hidden layers also increases the computational load for the network. The size of the neural network should be carefully chosen so that it is large enough to absorb all the information and yet small enough so that it can be easily trained.

In a comparison study between three different auto-regressive modeling techniques, a back propagation neural network was found to be the most appropriate. In these techniques [11], the model provided a prediction of a mentioned input parameter based on the regression of its previous values.

The flow chart of neural forecasting processing is generally used by which in figure 3.

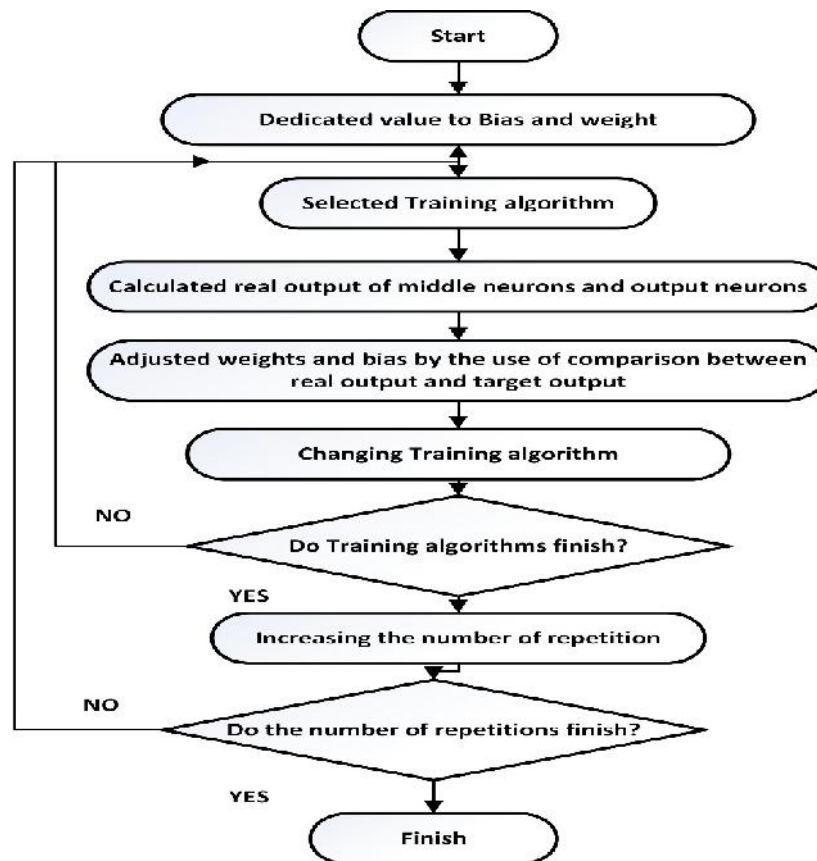


Fig.3. the flow chart of predication with ANN

SHEAR STRESS:

The source of AE in a hydrodynamic bearing, given that under normal operating conditions the hydrodynamic oil film is sufficiently large to ensure no asperity contact, can only be attributed to the friction within the fluid and the friction reaction of the fluid on the shaft. Such friction in fluid is attributed to the shear stress which is defined as the force per unit area exerted by a solid boundary on a fluid in motion in a direction on the local tangent plane [16]. The shear stress for a Newtonian fluid can be expressed as following equation:

Eq.4

$$\tau_w = \sim \frac{Uu}{y}$$

- ~ is the dynamic viscosity of the fluid;
- u is the velocity of the fluid along the boundary
- y is the height of the boundary.

In calculation of shear stress the inner cylinder (shaft) rotates at constant speed and the inner diameter at the bearing bush is assumed stationary. It is also assumed that the oil flow is laminar. To obtain an expression for velocity profile and shear

stress distribution a few assumption have been made, including:

- (1). Steady flow, which eliminates time variations in fluid properties and equations;
- (2) Temperature profile within the oil film is assumed constant;
- (3) Incompressible flow, simplifying the governing equations of fluid flow can;
- (4) Periphery symmetric flow;
- (5) No flow or variation of properties in the direction axial

The relationship between velocity profile, shear stress and film thickness were determined by Rook [17]

Eq.5

$$u_r = c_1 \frac{r}{2} + c_2 \frac{1}{r}$$

u_r is velocity profile, c_1, c_2 is boundary condition constant, r is radius of interest

Eq.6

$$\ddagger_{r''} = \sim \frac{2\check{S} R_2^2 1}{1 - \left(\frac{R_1}{R_2}\right)^2 h_0^2}$$

Where:

$\ddagger_{r''}$ —is shear stress distribution in the bearing (Pa);

R1—is dynamic viscosity(Pas);

m —is rotational speed(rads₋₁);

R2 —is shaft radius(mm);

O —is bearing radius(mm);

h0 —is film thickness (mm).

From the Eq.(6)theoretical estimates of shear stress were determined.

This shows a steep increase in shear stress levels between 750, 1500, 2500, 3500, and 4500 rpm after which levels remained relatively steady (750–1500rpm).From 2500 to 4500 rpm the levels of shear stress steadily decreased with increase speed. As is illustrated in Figure 4 , by increasing speed in a fix load, the shear stress was increased and then decreased ,this pattern has been forecasted by the application of MLP and as it can be depicted in below graph the real output which have been calculated by formula as real data and predicated output of MLP were completely compatible, the mentioned curve have been fitted with 0.001.

THE INFLUENCE OF PRESSURE AND TEMPERATURE

Reynolds generalized equation for the distribution of pressures of a fluid between plates has the following expression:

Eq.7

$$\begin{aligned} & \frac{\partial}{\partial x} \left(\frac{\dots h^3}{12} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\dots h^3}{12} \frac{\partial p}{\partial y} \right) = \\ & = \frac{\partial}{\partial x} \left(\frac{\dots h (u_a + u_b)}{2} \right) + \frac{\partial}{\partial y} \left(\frac{\dots h (\epsilon_a + \epsilon_b)}{2} \right) + \\ & \dots (w_a + w_b) - \dots u_a \frac{\partial h}{\partial x} - \dots \epsilon_a \frac{\partial h}{\partial y} + h \frac{\partial p}{\partial t} \end{aligned}$$

‘u’,‘v’,‘w’ : the coordinates of the velocity vector

‘μ’ : absolute viscosity of the fluid,

‘p’ : pressure

‘h’ : lubricant film thickness in the z-direction

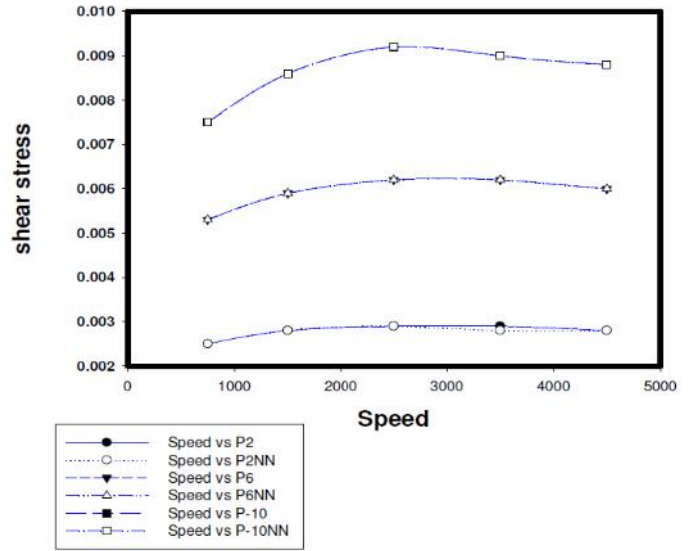


Fig.4. comparison between real out output and prediction output of shear stress via various loads and speed by Neural network

POWER LOSS

The friction factor is dimensionless parameter which is determined from the eccentricity ratio and Sommerfeld number of the bearing, and, is employed to calculate the power losses of the bearing [18]. The result presented in figure. 5 was similar for all three different viscosities investigated. The relationship between rotational speed and theoretical power losses on the bottom of the journal bearing was calculated as is shown in figure 5. Even though the friction factor reduces in value with increasing speed, the power losses increase with increasing speed.

This could be predicted by the use of MLP network. Thus the estimation of power losses has a direct relationship with speed this was considered in training algorithm in neural network and prognosticating of power loss was compatible in real graph. the levels of reduction in friction factor is significantly outweighed by increase in speed.

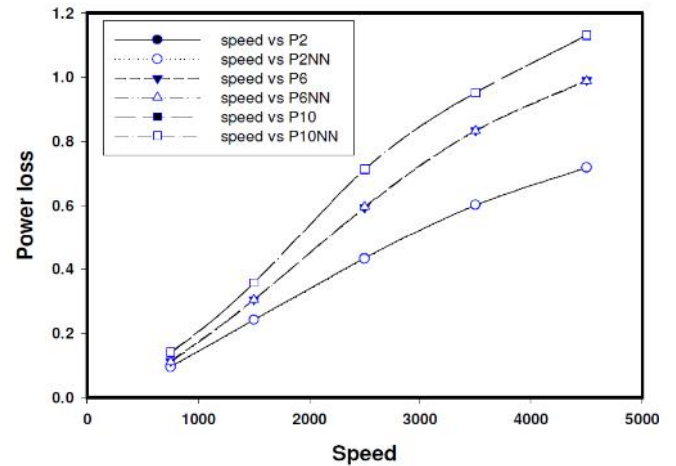


Fig.5 comparison between real out output and prediction output of power loss via various loads and speeds by Neural network

AE R.M.S

Faults in the bearing were detected through methods of processing the vibration and were then measured near the bearing. These methods were known as RMS value, crest factor analysis, kurtosis analysis and shock pulse counting. AE techniques are proving themselves to be a significant system of monitoring problems in the area of journal bearings. In 1983, Mathews reported that AE signals can be translated through four main parameters: RMS, Energy, Counts and Events. Counts are the number of times the value of an AE signal exceeds a threshold value. RMS is a statistical value to represent the dissipated energy (in Volts) that follows the equation;

Eq.8

$$AE_{rms} = \sqrt{1/N \sum_{i=1}^N AE_i^2}$$

In current work, AE levels increased with increasing power losses for all three different dynamic viscosities. However at any given speed and dynamic viscosity the variation in load caused an increase in AE levels though the levels were relatively small in relation to the increase in AE levels with increasing speed. AE will be generated from not only shear stress in the region of minimum film thickness but also from splashing of the oil and its motion within the top of the bearing. Therefore it is the total power losses and not the film

thickness or shear stress or friction factor that is most directly correlated to the measured AE levels.

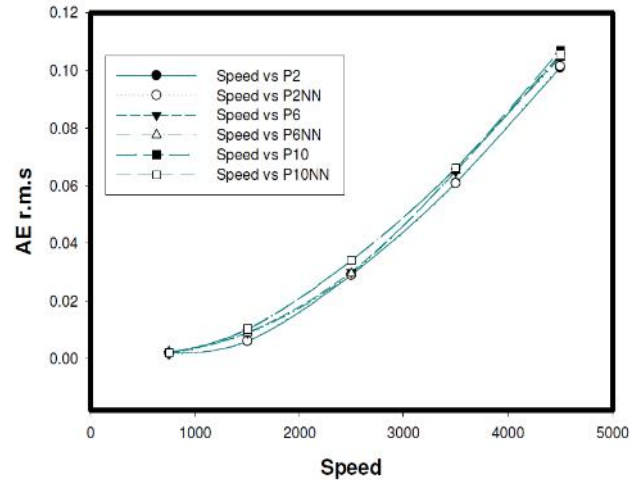


Fig.6 comparison between real out output and prediction output of AE r.m.s via various loads and speed by Neural network

As is illustrated in figure 6, this can be found that the predicated AE r.m.s was completely fitted with real AE r.m.s which are influenced by shear stress and power loss, the neural network can utilize the effect of shear stress and power loss in generating AE r.m.s and this feature can reflect and prognosticate the emerge of defects in rotary machine.

Output(Exp)			Output (NN)			Errors		
Total Power loss	Shear Stress	AE r.m.s	Total Power loss	Shear Stress	AE r.m.s	Total Power loss	Shear Stress	AE r.m.s
0.0963	0.003	0.002	0.0946	0.003	0.0018	1.80	0	11.11
0.2427	0.003	0.006	0.242	0.003	0.0061	0.29	0	-1.64
0.4347	0.003	0.029	0.4325	0.003	0.0291	0.51	0	-0.34
0.6015	0.003	0.061	0.6029	0.003	0.0609	-0.23	3.57	0.16
0.2303	0.004	0.002	0.2304	0.004	0.0019	-0.04	0	5.26
0.5281	0.005	0.008	0.5315	0.005	0.0075	-0.64	0	6.67
0.8288	0.005	0.03	0.8254	0.005	0.03	0.41	0	0.00
1.0697	0.004	0.05	1.0718	0.004	0.05	-0.20	0	0.00
0.2991	0.005	0.002	0.2982	0.005	0.0017	0.30	0	17.65
0.7711	0.006	0.008	0.7688	0.006	0.0085	0.30	0	-5.88
1.2804	0.006	0.033	1.2854	0.006	0.0328	-0.39	0	0.61
1.6224	0.006	0.061	1.6156	0.006	0.0611	0.42	0	-0.16

Table2. Comparison between real output and predicted Output (NN) and calculation of relevant Errors

the mentioned table which has been depicted above, is about experimental output which have been gathered from test-rig , the predicted outputs of AE r.m.s , shear stress and Power loss have been calculated by Neural network method , the errors can show that there is a little deviation of real data (about 0.001). Providing high accuracy of prediction can be applied for condition monitoring of difficult situation.

CONCLUSION

Journal bearing utilize for various application in our industries. However this is one of the immolated devices, it can reflect the various defects as soon as possible the defects have been occurred in our test-rig. Predicting defect, because of the presence of defects in this type of bearing, is highly appreciated in condition monitoring. In this current work,

multi-layer perceptron used to prognosticate the mechanicals and Acoustic emission output parameters. The precision of the mentioned network is about 0.001. Because of the high precision in prediction of output parameters, it can be utilized for predicting and interpolating of parameters which cannot be tested in real condition with lack of real test bed.

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